

Question

Evaluate the determinant $|A|$ associated with the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 1 & 1 \end{pmatrix}$

in the following ways:

- (a) Using the cofactor formula on the third row;
- (b) Using the cofactor formula on the second column;
- (c) By subtracting the second row from the first row, and then using some general properties of determinants.

Answer

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 1 & 1 \end{vmatrix}$$

- (a) Using cofactor formula on the third row

$$\det(A) = 1 \times (-3) + 1 \times 6 + 1 \times (-3) = 0$$

- (b) Using cofactor formula on the second column

$$\det(A) = 2 \times 2 + 5 \times (-2) + 1 \times 6 = 0$$

- (c) general properties (row 2 \rightarrow row 3 - row 1)

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 3 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 3 \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0 \text{ because two rows are identical.}$$