Question
Find the vector product $\mathbf{a} \times \mathbf{b}$ if:

(i) $\mathbf{a} = \mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$ $\mathbf{b} = 2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$

(ii) $\mathbf{a} = -\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$

(iii) $\mathbf{a} = -2\mathbf{i} + 4\mathbf{k}$ $\mathbf{b} = 3\mathbf{j} - 2\mathbf{k}$

What are the corresponding vector products $\mathbf{b} \times \mathbf{a}$.

Answer

(i)

$$\begin{aligned}
& \mathbf{a} \times \mathbf{b} \\
= & \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 4 \\ 2 & -2 & 3 \end{vmatrix} \\
= & \mathbf{i}(-2 \times 3) - \mathbf{i}(-2 \times 4) - \mathbf{j}(1 \times 3) \\
& +\mathbf{k}(1 \times -2) + \mathbf{j}(4 \times 2) - \mathbf{k}(2 \times -2) \\
= & -6\mathbf{i} + 8\mathbf{i} - 3\mathbf{j} - 2\mathbf{k} + 8\mathbf{j} + 4\mathbf{k} \\
= & 2\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}
\end{aligned}$$

[This should be perpendicular to both $\mathbf{a}$ and $\mathbf{b}$. Hence the dot product should be 0.

Check:

$\mathbf{a} \cdot (2\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}) = (1, -2, 4) \cdot (2, 5, 2) = 2 - 10 + 8 = 0\surd$

$\mathbf{b} \cdot (2\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}) = (2, -2, 3) \cdot (2, 5, 2) = 4 - 10 + 6 = 0\surd$]

(ii)

$$\begin{aligned}
& \mathbf{a} \times \mathbf{b} \\
= & \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 4 & -1 \\ 3 & 2 & 4 \end{vmatrix} \\
= & \mathbf{i}(4 \times 4) - \mathbf{i}(2 \times -1) - \mathbf{j}(-1 \times 4) \\
& +\mathbf{k}(-1 \times -2) + \mathbf{j}(3 \times -1) - \mathbf{k}(3 \times 4) \\
= & 16\mathbf{i} + 2\mathbf{i} + 4\mathbf{j} - 2\mathbf{k} - 3\mathbf{j} - 12\mathbf{k} \\
= & 18\mathbf{i} + \mathbf{j} - 14\mathbf{k}
\end{aligned}$$

[again check that $\mathbf{a} \cdot (18\mathbf{i} + \mathbf{j} - 14\mathbf{k}) = 0$ so they're perpendicular

$= (-1, 4, -1) \cdot (18, 1, -14)$
$= -18 + 4 + 14 = 0\surd$

also: $\mathbf{b} \cdot (18\mathbf{i} + \mathbf{j} - 14\mathbf{k}) = 0$

$= (3, 2, 4) \cdot (18, 1, -14)$
$= 54 + 2 - 56 = 0\surd$]

(iii)

$$\begin{aligned}
& \mathbf{a} \times \mathbf{b} \\
= & \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 0 & 4 \\ 0 & 3 & -2 \end{vmatrix} \\
= & \mathbf{i}(0 \times -2) - \mathbf{i}(3 \times 4) - \mathbf{j}(-2 \times -2) \\
& +\mathbf{k}(-2 \times 3) + \mathbf{j}(0 \times -2) - \mathbf{k}(0 \times 0) \\
= & -12\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}
\end{aligned}$$

[again check that $\mathbf{a} \cdot (-12\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}) = 0$ so they're perpendicular

$= (-2, 0, 4) \cdot (-12, -4, -6)$
$= 24 + 0 + -24 = 0\surd$

also: $\mathbf{b} \cdot (-12\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}) = 0$

$= (0, 3, -2) \cdot (-12, -4, -6)$
$= 0 - 12 + 12 = 0\surd$]

The corresponding vector products $\mathbf{b} \times \mathbf{a}$ are just the same as above $\times - 1$. Hence

(i) $(\mathbf{b} \times \mathbf{a}) = -12\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$

(ii) $(\mathbf{b} \times \mathbf{a}) = -18\mathbf{i} - \mathbf{j} + 14\mathbf{k}$

(iii) $(\mathbf{b} \times \mathbf{a}) = 12\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$