Question

The equations

$$\frac{3x+3}{2} = \frac{-2y+1}{7} = \frac{2z+6}{3}$$

determines a straight line. Express this in its equivalent parametric form and hence as a vector equation.

Answer

If $\dfrac{3x+3}{2} = \dfrac{-2y+1}{7} = \dfrac{2z+6}{3}$ we form the parametric equation by reversing the steps of Q5:

so set

$\dfrac{3x+3}{2} = \lambda,\ \dfrac{-2y+1}{7} = \lambda,\ \dfrac{2z+6}{3} = \lambda.$

$\Rightarrow \left\{ x = \dfrac{2\lambda-3}{3};\ y = \dfrac{1-7\lambda}{2};\ z = \dfrac{3\lambda-6}{2} \right\}$ <u>parametric equations</u>

Vector equation is again got by reversing the steps of Q5:

If $x = \dfrac{2\lambda-3}{3}$

$\Rightarrow$ $\mathbf{i}$ component of vector equation is $\dfrac{2\lambda}{3} - \dfrac{3}{3} = \dfrac{2\lambda}{3} - 1$

If $y = \dfrac{1-7\lambda}{2}$

$\Rightarrow$ $\mathbf{j}$ component of vector equation is $\dfrac{1}{2} - \dfrac{7}{2}\lambda$

If $z = \dfrac{3\lambda-6}{2}$

$\Rightarrow$ $\mathbf{k}$ component of vector equation is $\dfrac{3\lambda}{2} - 3$

Thus $\mathbf{r} = \left(\dfrac{2\lambda}{3} - 1\right)\mathbf{i} + \left(\dfrac{1}{2} - \dfrac{7}{2}\lambda\right)\mathbf{j} + \left(\dfrac{3\lambda}{2} - 3\right)\mathbf{k}$

or $\underline{\mathbf{r} = -\mathbf{i} + \dfrac{1}{2}\mathbf{j} - 3\mathbf{k} + \lambda\left(\dfrac{2}{3}\mathbf{i} - \dfrac{7}{2}\mathbf{j} + \dfrac{3}{2}\mathbf{k}\right)}$