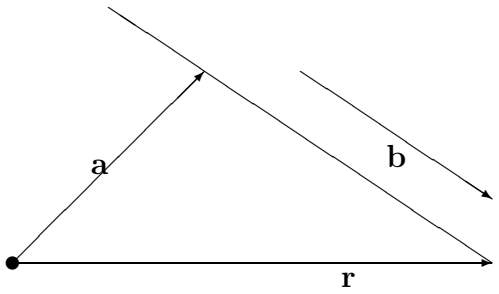
Question

Find the vector equation of the line through the point with position vector $\mathbf{a} = 2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ which is parallel to the vector $\mathbf{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}$. Determine the points corresponding to $\lambda = 3, 0, 2$ in resulting equation. Write down the parametric and cartesian equations of the line.

Answer



Vector equation: $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} = 2\mathbf{i} - \mathbf{j} - 3\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + \mathbf{k})$
$\underline{\lambda = -3}$: $\mathbf{r} = 2\mathbf{i} - \mathbf{j} - 3\mathbf{k} - 3(\mathbf{i} + \mathbf{j} + \mathbf{k}) = -\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}$
$\underline{\lambda = 0}$: $\mathbf{r} = 2\mathbf{i} - \mathbf{j} - 3\mathbf{k} + 0(\mathbf{i} + \mathbf{j} + \mathbf{k}) = 2\mathbf{i} - \mathbf{j} - 3\mathbf{k} = \mathbf{a}$
$\underline{\lambda = 2}$: $\mathbf{r} = 2\mathbf{i} - \mathbf{j} - 3\mathbf{k} + 2(\mathbf{i} + \mathbf{j} + \mathbf{k}) = 4\mathbf{i} + \mathbf{j} - \mathbf{k}$
Parametric equation:
$\mathbf{r} = 2\mathbf{i} - \mathbf{j} - 3\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + \mathbf{k})$
$= (2 + \lambda)\mathbf{i} + (\lambda - 1)\mathbf{j} + (\lambda - 3)\mathbf{k}$
General point is (x, y, z), so

$$\left\{ \begin{array}{l} x = 2 + \lambda \\ y = \lambda - 1 \\ z = \lambda - 3 \end{array} \right\}$$

Parametric equation
Cartesian equation: λ must be the same for each of the parametric equations.
Thus eliminating λ,
$x - 2 = \lambda,\ y + 1 = \lambda,\ z + 3 = \lambda \Rightarrow \underline{x - 2 = y + 1 = z + 3}$