Question
Find the line of intersection between the planes $x+2y+3z=6$ and $x+y+z=1$.

Answer

$$\left.\begin{array}{lrcl} (1) & x+2y+3z & = & 6 \\ (2) & x+y+z & = & 1 \end{array}\right\}$$ require line of intersection: 2 planes intersect in a line common to both planes
PICTURE

Take the two simultaneous equations away from each other to eliminate x:
$(1)-(2)$:
$x+2y+3z=6$
$\underline{x+\ y+\ z\ =1}$
$\quad y+2z=5$
Thus the planes intersect along $y=2z+5$ or $\underline{y=-2z+5}$.
But this doesn't tell us about the x dependence.
Thus we have to do another elimination, say of y by $2\times(2)-(1)$
$2x+2y+2z=\ 2$
$\underline{x+2y+3z=\ 6}$
$x \quad - \ z=-4$
Hence we have that

$$\begin{array}{rcll} y+2z & = & 5 & (3) \\ x-z & = & -4 & (4) \end{array}$$

Thus $x+4=z$ from (4)
and $-\left(\dfrac{y+5}{2}\right)=z$ from (3)
Remember a straight line in $3-D$ is given by

$$\frac{x-\alpha_1}{\beta_1}=\frac{y-\alpha_2}{\beta_2}=\frac{z-\alpha_3}{\beta_3}$$

for given $\alpha_i,\ \beta_i$
Thus we combine the above two equalities to get

$$\underline{x+4=-\left(\frac{y+5}{2}\right)=z}$$

as the equation of intersection.

Note that from questions above, we can write this in vector form:

$x + 4 = \lambda;\ -\left(\dfrac{y+5}{2}\right) = \lambda;\ z = -\lambda$

$\Rightarrow x = \lambda - 4;\ y = -5 - 2\lambda;\ z = \lambda$ parametric equation

$\Rightarrow \mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = (\lambda - 4)\mathbf{i} + (-5 - 2\lambda)\mathbf{j} + \lambda\mathbf{k}$

$\Rightarrow \underline{\mathbf{r} = -4\mathbf{i} - 5\mathbf{j} + \lambda(\mathbf{i} - 2\mathbf{j} + \mathbf{k})}$