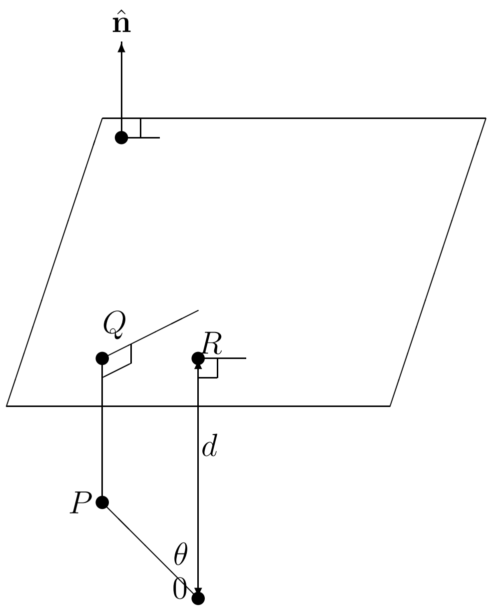
Question
Find the distance of the point(1,1,1) from the plane $x + 2y + 3z = 6$. Write down the vector equation of the plane.

Answer



Let **OP** be **i**, **j**, **k** and **PQ** be the perpendicular distance from P to the plane.
We require $|\mathbf{PQ}|$.
PQ is in the direction of **n** the normal to the plane, by definition.
Now the equation of the plane is

$$x + 2y + 3z = 6$$

which can be rewritten as

$$(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \cdot (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) = 6$$

$\mathbf{n}$ is thus

$$\begin{aligned}
\frac{(\mathbf{i}+2\mathbf{j}+3\mathbf{k})}{|(\mathbf{i}+2\mathbf{j}+3\mathbf{k})|} &= \frac{(\mathbf{i}+2\mathbf{j}+3\mathbf{k})}{\sqrt{1+4+9}} \\
&= \frac{1}{\sqrt{14}}\mathbf{i}+\frac{2}{\sqrt{14}}\mathbf{j}+\frac{3}{\sqrt{14}}\mathbf{k}
\end{aligned}$$

Thus $\mathbf{r}\cdot\left(\frac{1}{\sqrt{14}}\mathbf{i}+\frac{2}{\sqrt{14}}\mathbf{j}+\frac{3}{\sqrt{14}}\mathbf{k}\right)=\frac{6}{\sqrt{14}}$
A vector equation of the plane.
This, $\left(\frac{6}{\sqrt{14}}\right)$, is the perpendicular distance of 0 from the plane $d=|\mathbf{OR}|$.
Clearly $|\mathbf{PQ}|=|\mathbf{OR}|-|\mathbf{OP}|\cos\theta$
What is $|\mathbf{OP}|\cos\theta$?

$$|\mathbf{OP}|\cdot\hat{\mathbf{n}}=|\mathbf{OP}||\hat{\mathbf{n}}|\cos\theta=|\mathbf{OP}|\cos\theta$$

Thus

$$\begin{aligned}
|\mathbf{PQ}| &= |\mathbf{OR}|-\mathbf{OP}\cdot\hat{\mathbf{n}} \\
&= d-(\mathbf{OP}\cdot\hat{\mathbf{n}}) \\
&= \frac{6}{\sqrt{14}}-(\mathbf{i}+\mathbf{j}+\mathbf{k})\cdot\left(\frac{1}{\sqrt{14}}\mathbf{i}+\frac{2}{\sqrt{14}}\mathbf{j}+\frac{3}{\sqrt{14}}\mathbf{k}\right) \\
&= \frac{6}{\sqrt{14}}-\left(\frac{1}{\sqrt{14}}\mathbf{i}+\frac{2}{\sqrt{14}}\mathbf{j}+\frac{3}{\sqrt{14}}\mathbf{k}\right) \\
&= \underline{0}!!!
\end{aligned}$$

Thus it looks like (1,1,1) is in the plane. Ooops it is!
Since if

$$\begin{aligned}
x+2y+3z &= 6 \\
\Rightarrow 1+2+3 &= 6 \\
\Rightarrow 6 &= 6
\end{aligned}$$

(if $(x,y,z)=(1,1,1)$)
Thus the distance of the point (1,1,1) from the plane is 0, since it lies in it!!!