**Question**
Evaluate the triple scalar products $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$ and $\mathbf{b} \times \mathbf{a} \cdot \mathbf{c}$ given that:

**(i)** $\mathbf{a} = 2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ $\mathbf{b} = 3\mathbf{k}$ $\mathbf{c} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

**(ii)** $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$ $\mathbf{b} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$ $\mathbf{c} = 4\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

**Answer**

$$\begin{aligned} \mathbf{a} \cdot \mathbf{b} \times \mathbf{c} &= (a_1\mathbf{a} + a_2\mathbf{a} + a_3\mathbf{a}) \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \\ &= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \end{aligned}$$

(think about it!)

**(i)**

$$\begin{aligned} \mathbf{a} \cdot \mathbf{b} \times \mathbf{c} &= \begin{vmatrix} 2 & -1 & -3 \\ 0 & 0 & 3 \\ 1 & 2 & 2 \end{vmatrix} \\ &= (2 \times 0 \times 2) - (2 \times 2 \times 3) - (0 \times -1 \times 2) \\ &\quad +(0 \times 2 \times -3) + (1 \times -1 \times 3) - (1 \times 0 \times -3) \\ &= 0 - 12 - 0 + 0 - 3 + 0 \\ &= \underline{-15} \end{aligned}$$

Now

$$\begin{aligned} (\mathbf{b} \times \mathbf{a}) \cdot \mathbf{c} &= -(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} \\ &= -\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) \\ &= \underline{+15} \end{aligned}$$

by relevance to the above determinant.

**(ii)**

$$\begin{aligned}
& \mathbf{a} \cdot \mathbf{b} \times \mathbf{c} \\
= & \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 4 & 2 & 2 \end{vmatrix} \\
= & (1 \times 1 \times 2) - (1 \times 2 \times 1) - (2 \times 2 \times 2) \\
& +(2 \times 2 \times 1) + (4 \times 2 \times 1) - (4 \times 1 \times 1) \\
= & 2 - 2 - 8 + 4 + 8 - 4 \\
= & \underline{0}
\end{aligned}$$

Hence $(\mathbf{b} \times \mathbf{a}) \cdot \mathbf{c} = -(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = -0 = \underline{0}$