

**Question**

Express the following vectors as the product of a scalar and a unit vector.

(i)  $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$

(ii)  $\mathbf{a} = 3\mathbf{i} - 3\mathbf{j} + \mathbf{k}$

(iii)  $\mathbf{a} = \frac{-\sqrt{71}}{9}\mathbf{i} - \frac{1}{3}\mathbf{j} - \frac{1}{9}\mathbf{k}$

**Answer**

(i)  $|\mathbf{a}| = \sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{4 + 1 + 9} = \sqrt{14}$

$$\text{Therefore } \mathbf{a} = \hat{\mathbf{a}}|\mathbf{a}| = \sqrt{14} \left( \frac{2\mathbf{i}}{\sqrt{14}} - \frac{\mathbf{j}}{\sqrt{14}} + 3\frac{\mathbf{k}}{\sqrt{14}} \right)$$
$$|\hat{\mathbf{a}}| = 1$$

(ii)  $|\mathbf{a}| = \sqrt{3^2 + (-3)^2 + 1^2} = \sqrt{19}$

$$\text{Therefore } \mathbf{a} = \hat{\mathbf{a}}|\mathbf{a}| = \sqrt{19} \left( \frac{3\mathbf{i}}{\sqrt{19}} - 3\frac{\mathbf{j}}{\sqrt{19}} + \frac{\mathbf{k}}{\sqrt{19}} \right)$$

(iii)  $|\mathbf{a}| = \sqrt{\frac{71}{81} + \frac{1}{9} + \frac{1}{81}} = \frac{\sqrt{81}}{9} = 1$

Thus  $\mathbf{a}$  is already a unit vector so  $\mathbf{a} = \hat{\mathbf{a}} = -\frac{\sqrt{71}}{9}\mathbf{i} - \frac{1}{3}\mathbf{j} - \frac{1}{9}\mathbf{k}$