Question

Which of the following pairs of vector are parallel and which are anti-parallel?

(i) $\mathbf{a} = \mathbf{i} - 3\mathbf{j} + \mathbf{k}$ $\mathbf{b} = -4\mathbf{i} + 12\mathbf{j} - 4\mathbf{k}$

(ii) $\mathbf{a} = -2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ $\mathbf{b} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$

(iii) $\mathbf{a} = 4\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ $\mathbf{b} = 8\mathbf{i} - 2\mathbf{j} - 6\mathbf{k}$

(iv) $\mathbf{a} = \mathbf{i} + 7\mathbf{j} + \mathbf{k}$ $\mathbf{b} = 3\mathbf{i} + 21\mathbf{j} + 3\mathbf{k}$

Answer

Can be written as $\lambda\mathbf{a}$ where λ is a scalar. If so, then if $\lambda > 0$ the vectors are parallel. If so but $\lambda < 0$ then they're anti-parallel. If <u>not</u> then they're not parallel.

(i) $\mathbf{a} = \mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{b} = -4\mathbf{i} + 12\mathbf{j} - 4\mathbf{k} = -4(\mathbf{j} - 3\mathbf{k} + \mathbf{j}) \Rightarrow \lambda = -4 \Rightarrow$ anti-parallel.

(ii) $\mathbf{a} = -(2\mathbf{i} - 3\mathbf{j} + \mathbf{k}) = -\mathbf{b} \Rightarrow$ anti-parallel.

(iii) $\mathbf{b} = 8\mathbf{i} - 2\mathbf{j} - 6\mathbf{k} = 2(4\mathbf{i} - \mathbf{j} - 3\mathbf{k}) = 2\mathbf{a} \Rightarrow$ parallel

(iv) $\mathbf{a} = \mathbf{i} + 7\mathbf{j} + \mathbf{k}$, $\mathbf{b} = 3\mathbf{i} + 21\mathbf{j} + 3\mathbf{k} = 3(\mathbf{j} + 7\mathbf{j} + \mathbf{k}) = 3\mathbf{a} \Rightarrow$ parallel