Question

Assume that $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi b}} e^{-\frac{1}{2b^2}(y-a)^2}\,dy = 1$ where $b > 0$.

Evaluate the integral $\int_0^{\infty} e^{-3y^2}\,dy$.

Answer

We know that

$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi} b} e^{-\frac{(y-a)^2}{2b^2}}\,dy = 1$

Therefore $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi} b} e^{-\dfrac{y^2}{2b^2}}\,dy = 1$

i.e. $2\int_0^{\infty} \frac{1}{\sqrt{2\pi} b} e^{-\frac{y^2}{2b^2}}\,dy = 1$

i.e. $\int_0^{\infty} e^{-\frac{y^2}{2b^2}}\,dy = \frac{\sqrt{2\pi} b}{2}$

Let $\frac{1}{2b^2} = 3 \Rightarrow b^2 = \frac{1}{6} \Rightarrow b = \sqrt{\frac{1}{6}}$

Therefore $\int_0^{\infty} e^{-3y^2}\,dy = \frac{\sqrt{2\pi}}{2}\sqrt{\frac{1}{6}} = \frac{\sqrt{\pi}}{2\sqrt{3}}$