QUESTION A continuous random variable X is uniformally distributed in the interval $-1 \leq x \leq 1$. Find E(X) and Var(X). The random variable Y is defined by $Y = X^2$, Use the cdf of X to show that $P(Y \leq y) = \sqrt{Y},\ o \leq y \leq 1$ and obtain the pdf of Y. Hence or otherwise evaluate E(Y) and Var(Y).

ANSWER $f(x) = \frac{1}{2} \quad -1 \leq x \leq 1$
$E(X) = 0(\text{ by symmetry}) = \int_{-1}^{1} \frac{1}{2}x\,dx$
$E(X^2) = Var(X) = \int_{-1}^{1} \frac{1}{2}x^2\,dx = [\frac{1}{6}x^3]_{-1}^{1} = \frac{1}{3}$

$$\begin{aligned} F(x) &= \int_{-1}^{x} \frac{1}{2}\,ds \\ &= \frac{1}{2}(x+1) \\ &= P(-\sqrt{y} \leq X \leq \sqrt{y}) \\ &= F(\sqrt{y}) - F(-\sqrt{y}) = \sqrt{y}, \quad 0 \leq y \leq 1 \end{aligned}$$

$g(y) = \frac{d}{dy}P(Y \leq y) = \frac{1}{2\sqrt{y}}, \quad 0 \leq y \leq 1$
$E(Y) = E(X^2) = \frac{1}{3} = \int_0^1 \frac{1}{2}\sqrt{y}\,dy$
$E(Y^2) = EX(X^4) = \frac{1}{5} = \int_0^1 \frac{1}{2}y^{\frac{3}{2}}\,dy$ Therefore Var(Y)$= \frac{1}{5} - \frac{1}{9} = \frac{4}{45}$