QUESTION The length of a certain type of battery is normally distributed with mean 5.0cm and standard deviation .05cm. Find the probability that such a battery has a length between 4.92 and 5.08 cm.
tubes are manufactured to contain 4 such batteries. 95% of the tubes have lengths greater than 20.9 and 10% have lengths greater than 21.6cm. Assuming that the lengths of the tubes are also normally distributed, find the mean and standard deviation of the lengths correct to two decimal places.
If tubes and batteries are chosen independently, find the probability that a tube will contain 4 batteries with at least 0.75cm to spare.

ANSWER $B \sim N(5.0, 0.5^2)$

$$\begin{aligned} P(4.92 < B < 5.08) &= \Phi(\frac{5.08 - 5.0}{0.05}) - \Phi(\frac{4.92 - 5.0}{.05}) \\ &= \Phi(1.6) - \Phi(-1.6) = 0.9452 - (1 - 0.9452) \approx 0.89 \end{aligned}$$

$95\% > 20.9 \rightarrow 20.9 = \mu - 1.6449\sigma$
$10\% > 21.6 \rightarrow 21.6 = \mu + 1.2816\sigma$
Subtracting the two equations gives $0.7 = 2.9265\sigma$, $\sigma = 0.2392 = 0.24$ to 2 d.p.Substituing in either equation gives that $\mu = 21.29$ to 2 d.p.
Distribution of $B_1 + B_2 + B_3 + B_4 \sim N(20.0, 4 \times 0.05^2) = N(20.0, 0.01)$
$T - (B_1 + B_2 + B_3 + B_4) \sim N(1.29, 0.24^2 + 0.01)$

$$\begin{aligned} P(T - (B_1 + B_2 + B_3 + B_4) > 0.75) &= 1 - \Phi(\frac{0.75 - 1.29}{sqrt 0.0676}) \\ &= 1 - \Phi(-\frac{0.54}{0.26}) \\ &= 1 - \Phi(-2.077) = \Phi(2.077) \\ &= 0.981 \end{aligned}$$