

QUESTION The time to failure of a new type of lightbulb is thought to have an exponential distribution.

If the reliability of this type of lightbulb at 10.5 weeks is 0.9 find the reliability at 10 weeks. (Note that reliability is defined as the probability that the article will not have failed by the specified time).

100 bulbs of this type are put in a new shop. All the bulbs that have failed are replaced at 20 week intervals and none are replaced at other times. If R is the number of bulbs that have to be replaced, find the mean and variance of R. Explain why this result will hold for **any** such interval and not just the first,

ANSWER $f(x) = \lambda e^{-\lambda x}$ $F(x) = 1 - e^{-\lambda x}$ reliability at $x = P(x > x) = 1 - F(x) = e^{-\lambda x}$
therefore

$$\begin{aligned} e^{-\lambda 10.5} &= 0.9 \\ 10.5\lambda &= \ln \frac{10}{9} \\ \rightarrow \lambda &= 0.010034 \end{aligned}$$

Reliability at 20 weeks $= e^{-20\lambda} = 0.818$

R is the number replaced at the end of twenty weeks. $R \sim B(100, 0.182)$,
 $n=100$ $p=0.182$.

$$\mu = np = 100 \times 0.182 = 18.2$$

$$\sigma^2 = npq = 100 \times 0.182 \times 0.818 = 14.8$$

'No memory property' means it applies to any interval, since surviving bulbs 'start again'.