QUESTION A good model for the variation, from item to item, of a quality characteristic of a certain manufactured product is a random variable X with probability density function $f(x) = \frac{2x}{\lambda^2},\ 0 \leq x \leq \lambda$. Each of the manufactured items is tested and items for which $X > 1$, where $0 < 1 < \lambda$, are passed and the rest are rejected. The cost of a rejected item is $c = a\lambda + b$ and the profit on a passed item is $C - c$. The parameter λ can be adjusted to any desired value. Find λ such that the expected profit is maximised.

ANSWER $f(x) = \frac{2x}{\lambda^2},\ 0 \leq x \leq \lambda$

$$\begin{aligned}
\text{Profit} &= (C-c) \text{ if } X > L \\
&= -c \text{ if } X \leq L \\
E(\text{Profit}) &= (C-c)P(X > L) - cP(X \leq L) \\
&= CP(X > L) - c
\end{aligned}$$

$P(X > L) = \int_L^\lambda \frac{2x}{\lambda^2}\,dx = [\frac{x^2}{\lambda^2}]_L^\lambda = 1 - \frac{L^2}{\lambda^2}$
$E(\text{profit}) = C(1 - \frac{L^2}{\lambda^2}) - (a\lambda + b)$
$\frac{dE(\text{profit})}{d\lambda} = \frac{2cL^2}{\lambda^3} - a = 0$ when $\lambda^3 = \frac{2CL^2}{a}$

$$\lambda = \sqrt[3]{\frac{2CL^2}{a}}$$

Check that $\frac{d^2E(\text{profit})}{d\lambda^2} < 0$ to confirm that this is the maximum.