QUESTION The variable X has pdf $f(x) = \frac{1}{8}(6-x) \quad 2 \le x \le 6$. A sample of two values of X is taken. Denoting the lesser of the two values by Y, use the cdf of X to write down the cdf of Y. Obtain the pdf of Y and the mean of Y. Show that its median is approximately 2.64.

ANSWER $f(x) = \frac{1}{8}(6-x) \quad 2 \le x \le 6$
$Y = \min(X_1, X_2)$ where X_1 andX_2 are independent.
$P(Y > y) - P(X_1 \text{ and } X_2 > y) = P(X_1 > y)P(X_2 > y) = [1 - F(y)]^2$
$F(y) = \int_2^y \frac{1}{8}(6-x)\,dx = [-\frac{1}{8}(6-x)^2\frac{1}{2}]_2^y = 1 - \frac{1}{16}(6-y)^2$
$P(Y > y) = \frac{1}{16^2}(36 - 12y + y^2)^2 = \frac{1}{64}(18 - 6y + \frac{y^2}{1})^2$
$F_Y(y) = 1 - \frac{1}{64}(19 - 6y + \frac{y^2}{2})^2$
$f_Y(y) = \frac{1}{32}(18 - 6y + \frac{y^2}{2})(6-y), \quad 2 \le y \le 6$

$$\begin{aligned}
\mu_y &= \int_2^6 \frac{y}{32}(6-y)(18 - 6y + \frac{y^2}{2})\,dy \\
&= \frac{1}{32}\int_2^6 (108y - 36y^2 + 3y^3 - 18y^2 + 6Y^3 - \frac{y^4}{2})\,dy \\
&= \frac{1}{32}[54y^2 - 18y^3 + \frac{9}{4}y^4 - \frac{y^5}{10}]_2^6 \\
&= \frac{1}{32}[1944.4 - 104.8] = 2.8
\end{aligned}$$

To find median M need to find when cdf=$\frac{1}{2}$, so $\frac{1}{64}(18 - 6y + \frac{y^2}{2})^2 = \frac{1}{2} \Rightarrow (18 - 6y + \frac{y^2}{2})^2 = 32$

y	$(18 - 6y + \frac{y^2}{2})^2$
2.5	37.52
2.6	33.41
2.7	29.65
2.63	32.42
2.635	32.05
2.64	31.86

Hence we take M to be 2.64.