QUESTION

(i) Find gcd(16169,22747).

(ii) Find all the integral solutions, x and y, to the linear Diophantine equation

$$16169x + 22747y = 69.$$

ANSWER

(i) We use the Euclidean algorithm.

$$\begin{aligned}
22747 &= 1 \times 16169 + 6578 \\
16169 &= 13156 + 3013 = 2 \times 6578 + 3013 \\
6578 &= 6026 + 552 = 2 \times 3013 + 552 \\
3013 &= 2760 + 253 = 5 \times 552 + 253 \\
552 &= 2 \times 253 + 46 \\
253 &= 5 \times 46 + 23 \\
46 &= 2 \times 23
\end{aligned}$$

so that $\gcd(16169, 22747) = 23$

(ii) To solve this we must first observe that $69 = 3 \times 23$ so that there exists an infinite number of solutions. Next we must find one.

From the Euclidean algorithm in (i)

$$\begin{aligned}
23 &= 253 - 5 \times 46 \\
&= 253 - 5 \times (552 - 2 \times 253) \\
&= 11 \times 253 - 5 \times 552 \\
&= 11 \times (3013 - 5 \times 552) - 5 \times 552 \\
&= 11 \times 3013 - 60 \times 552 \\
&= 11 \times 3013 - 60 \times (6578 - 2 \times 3013) \\
&= 131 \times 3013 - 60 \times 6578 \\
&= 131 \times (16169 - 2 \times 6578) - 60 \times 6578 \\
&= 131 \times 16169 - 322 \times 6578 \\
&= 131 \times 16169 - 322 \times (22747 - 16169) \\
&= 453 \times 16169 - 322 \times 22747
\end{aligned}$$

Hence one solution is $x = 3 \times 453,\ y = -3 \times 332$. Therefore the general solution is

$$x = 3 + 3 \times \left(\frac{22747n}{23}\right),\ y = -3 \times 332 - 3 \times \left(\frac{16169n}{23}\right)$$

where n is an arbitrary integer.