Question
Prove that the Lebesgue measure of the interval $[a, b]$ is $(b - a)$ [in $\mathbf{R}$].

Answer
We shall use open covers by intervals to generate m^*. Let $\{R_i\}$ be an open cover of $[a, b]$. There is a finite subcover $\{R_1, \cdots R_n\}$ which we may suppose ordered so that $R_i = [a_i, b_i]$ and $a_1 \leq a_2 \leq \cdots \leq a_n$, also $a_1 < a$ and $a_n > b$, and so that no interval is entirely contained within another, then $b_i \leq b_j$ for $i < j$, for otherwise $(a_i, b_i) \supseteq (a_j, b_j)$
also $b_i > a_{i+1}$ for otherwise the point $\dfrac{b_i + a_{i+1}}{2}$ is not covered by the intervals.
Therefore $\sum |R_i| = \sum (b_i - a_i)$
$= b_n(-a_n + b_{n-1})(-a_{n-1} + \cdots) + b_1 - a_1$
$\geq b_n - a_1 \geq b - a$
Therefore $m^*([a, b]) \geq b - a$ Cover $[a, b]$ by $[a, b]$
Therefore $m^*([a, b]) = b - a$