Question

(a) Are the two lines $x = 3y + 7,\ \ y = z - 1$ and $x - 1 = 3y - 5 = 3z + 1$ parallel? (Give a reason.)

(b) At what point does the line $\mathbf{r} = \frac{1}{2}\mathbf{i} + u\mathbf{j} + \mathbf{k}\ \ u\epsilon\mathbf{R}$ cut the ellipse $\mathbf{r} = \sin t\mathbf{i} + \cos t\mathbf{j} + 2\sin t\mathbf{k},\ \ t\epsilon\mathbf{R}$

(c) What surface does the equation $\mathbf{r} = 4(\cos u\mathbf{i} + \sin u\mathbf{j}) + v\mathbf{k},\ \ u, v\epsilon\mathbf{R}$?

(d) Given a geometrical interpretation of the equation $|\mathbf{r} - \mathbf{a}|^2 = f^2$ when $\mathbf{a}$ is a given vector and f is a constant.

Answer

(a) Put the lines is standard form

$$\frac{x}{1} = \frac{y + \frac{7}{3}}{\frac{1}{3}} = \frac{z + 2}{\frac{1}{3}}\ :\ \frac{x - 1}{1} = \frac{y - \frac{5}{3}}{\frac{1}{3}} = \frac{z + \frac{1}{3}}{\frac{1}{3}}$$

They are parallel. Both have direction vector $\left(1, \dfrac{1}{3}, \dfrac{1}{3}\right)$

(b) $\mathbf{r} = \frac{1}{2}\mathbf{i} + u\mathbf{j} + \mathbf{k}$ meets

$\mathbf{r} = \sin t\mathbf{i} + \cos t\mathbf{j} + 2\sin t\mathbf{k}$

Where $\sin t = \frac{1}{2}$ and $\cos t = u$

So $\cos t = \pm\frac{\sqrt{3}}{2}$

So the line cuts the ellipse at two points $\left(\dfrac{1}{2}, \pm\dfrac{\sqrt{3}}{2}, 1\right)$

(c) $\mathbf{r} = 4(\cos u\mathbf{i} + \sin u\mathbf{j}) + v\mathbf{k}$ represents a cylinder centred on the z-axis with radius 4.

(d) $|\mathbf{r} - \mathbf{a}|^2 = f^2$ represents a sphere, centre $\mathbf{a}$ radius $|f|$