

Question

(a) For $\mathbf{c} = 5\mathbf{a} - \mathbf{b}$ and $\mathbf{d} = 3\mathbf{a} + 2\mathbf{b}$ find $\mathbf{c} \cdot \mathbf{d}$ when

(i) $\mathbf{a}$ and $\mathbf{b}$ are unit vectors at an angle $\frac{\pi}{4}$

(ii) $\mathbf{a}$ and $\mathbf{b}$ are perpendicular with $|\mathbf{b}| = 2|\mathbf{a}| = 2$.

(b) Evaluate $|\mathbf{c} \cdot \mathbf{d}|$ when $\mathbf{c} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{d} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$

Answer

(a) (i)

$$\begin{aligned}\mathbf{c} \cdot \mathbf{d} &= (5\mathbf{a} - \mathbf{b}) \cdot (3\mathbf{a} + 2\mathbf{b}) \\ &= 15\mathbf{a} \cdot \mathbf{a} + 10\mathbf{a} \cdot \mathbf{b} - 3\mathbf{a} \cdot \mathbf{b} - 2\mathbf{b} \cdot \mathbf{b} \\ &= 13 + \frac{7}{\sqrt{2}}\end{aligned}$$

(ii)

$$\begin{aligned}\mathbf{c} \cdot \mathbf{d} &= 15\mathbf{a} \cdot \mathbf{a} + 7\mathbf{a} \cdot \mathbf{b} - 2\mathbf{b} \cdot \mathbf{b} \\ &= 15|\mathbf{a}|^2 - 2|\mathbf{b}|^2 \\ &= 15 - 8 \\ &= 7\end{aligned}$$

(b) $\mathbf{c} = (1, 2, 3)$ $\mathbf{d} = (2, -1, -1)$

$$\mathbf{c} \cdot \mathbf{d} = 2 - 2 - 3 = -3 \text{ so } |\mathbf{c} \cdot \mathbf{d}| = 3$$