Question

(a) The position vectors of the points P and Q are

$$\mathbf{a} + 3\mathbf{b} - \mathbf{c} \quad \text{and} \quad 3\mathbf{a} - \mathbf{c}$$

Find $\vec{PQ}$ and $\vec{OM}$, where M is the midpoint of PQ, in terms of $\mathbf{a}, \mathbf{b}, \mathbf{c}$.

(b) For $\mathbf{d} = \mathbf{a} + \mathbf{b}$ and $\mathbf{e} = 3\mathbf{a} + \mathbf{b} - \mathbf{4c}$ determine scalars p and q such that

$$p\mathbf{d} + q\mathbf{e} = 10\mathbf{a} + 10\mathbf{b} - 12\mathbf{c}$$

when $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non-coplanar vectors.

Answer

(a) $\mathbf{p} = \mathbf{a} + 3\mathbf{b} - \mathbf{c}$ $\qquad$ $\mathbf{q} = 3\mathbf{a} - \mathbf{c}$

$\vec{PQ} = \mathbf{q} - \mathbf{p} = 2\mathbf{a} - 3\mathbf{b}$

$\vec{OM} = \frac{1}{2}(\mathbf{p} + \mathbf{q}) = 2\mathbf{a} + \frac{3}{2}\mathbf{b} - \mathbf{c}$

(b)

$$\begin{aligned} p\mathbf{d} + q\mathbf{e} &= p\mathbf{a} + \mathbf{b} + q(3\mathbf{a} + \mathbf{b} - \mathbf{4c}) \\ &= (p + 3q)\mathbf{a} + (p + q)\mathbf{b} - 4q\mathbf{c} \\ &= 16\mathbf{a} + 10\mathbf{b} - 12\mathbf{c} \end{aligned}$$

So $q = 3$, $p = 7$ as $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, are independent.