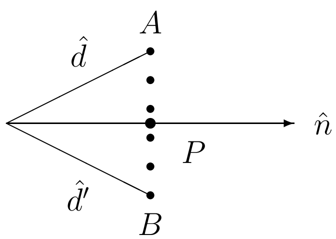
Question

The unit vector $\hat{\mathbf{n}}$ is along the bisector of the angle between the two unit vectors $\hat{\mathbf{d}}$ and $\hat{\mathbf{d}}'$. Show that

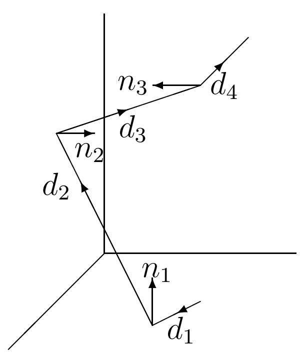
$$2(\hat{\mathbf{n}} \cdot \hat{\mathbf{d}})\hat{\mathbf{n}} = \hat{\mathbf{d}} + \hat{\mathbf{d}}'.$$

Hence prove that a ray of light emerges parallel to itself after successive reflections in each of three mutually perpendicular mirrors.

Answer



P is the mid point of AB So $(n \cdot d)\hat{n} = \dfrac{\mathbf{d}+\mathbf{d'}}{2} \Rightarrow 2(n \cdot d)\hat{n} = d + d'$



$$\begin{aligned} d_2 + (-d_1) &= 2(n_1 \cdot d_2)n_1 &(1)\\ d_3 + (-d_2) &= 2(n_1 \cdot d_3)n_2 &(2)\\ d_4 + (-d_3) &= 2(n_3 \cdot d_4)n_3 &(3) \end{aligned}$$

Adding (1), (2), (3)

$$d_4 - d_1 = 2(n_1 \cdot d_2)n_1 + 2(n_2 \cdot d_3)n_2 + 2(n_3 \cdot d_4)n_3 \qquad (4)$$

From (3) $n_2 \cdot d_4 - n_2 \cdot d_3 = 0$ since $n_2 \cdot n_3 = 0$
From (2) $n_1 \cdot d_3 - n_1 \cdot d_2 = 0$
From(3) $n_1 \cdot d_4 - n_1 \cdot d_3 = 0$ So $n_1 \cdot d_4 = n_1 \cdot d_2$
So (4) becomes

$$d_4 - d_1 = 2(n_1 \cdot d_4)N - 1 + 2(n_2 \cdot d_4)n_2 + 2(n_3 \cdot d_4)n_3 = 2d_4$$

So

$$d_4 = -d_1$$