Question

(a) For $\mathbf{c} = 5\mathbf{a} - \mathbf{b}$ and $\mathbf{d} = 3\mathbf{a} + 2\mathbf{b}$ find $\mathbf{c} \times \mathbf{d}$ when

(i) $\mathbf{a}$ and $\mathbf{b}$ are unit vectors at an angle $\frac{\pi}{4}$

(ii) $\mathbf{a}$ and $\mathbf{b}$ are perpendicular with $|\mathbf{b}| = 2|\mathbf{a}| = 2$.

(b) Evaluate $|\mathbf{c} \cdot \mathbf{d}|$ when $\mathbf{c} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{d} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$

Answer

(a) $\mathbf{c} = 5\mathbf{a} - \mathbf{b}$ and $\mathbf{d} = 3\mathbf{a} + 2\mathbf{b}$

$$\begin{aligned} c \times d &= (5\mathbf{a} - \mathbf{b}) \times (3\mathbf{a} + 2\mathbf{b}) \\ &= 15\mathbf{a} \times \mathbf{a} + 10\mathbf{a} \times \mathbf{b} - 3\mathbf{b} \times \mathbf{a} - 2\mathbf{b} \times \mathbf{b} \\ &= 13\mathbf{a} \times \mathbf{b} \end{aligned}$$

(i) $13\mathbf{a} \times \mathbf{b} = 13|a||b| \sin \frac{\pi}{4} \hat{n} = \frac{13}{\sqrt{2}} \hat{n}$

(ii) $13\mathbf{a} \times \mathbf{b} = 13|a||b| \sin \frac{\pi}{2} \hat{n} = 26\hat{n}$

(b) $|c \times d| = (1, 7, -5) \quad |c \times d| = 5\sqrt{3}$