Question

Two lines are given in space by the following equations.

$$\frac{x}{2} = \frac{y-1}{2} = z; \quad x+1 = y-2 = \frac{z+4}{2}$$

Find the equations of the following planes

(i) The plane containing the first line and parallel to the second line

(ii) The plane containing the second line and parallel to the first line

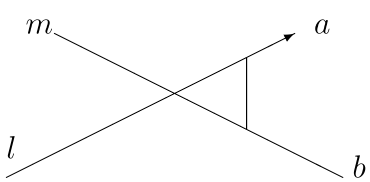
(iii) The plane containing the first line and passing through the origin

(iv) The plane containing the first line and the common perpendicular

(v) The plane containing the second line and the common perpendicular

(vi) The angle between the planes (iv) and (v).

Answer



(i)(ii) The plane containing l and that is parallel to m and the plane containing m and that is parallel to l both have $\mathbf{a} \times \mathbf{b}$ as a normal vector.

$(2,3,1) \times (1,1,2) = (3,-3,0)$

So the equation of plane (i) is $x - y = -19$ (contains (0,1,0))

So the equation of plane (ii) is $x - y = -3$ (contains (-1,2,-4))

(iii) Let the equation be $ax + by + cz = 0$

It contains the first line and so contains $(0,1,0)$. Thus $b = 0$.

Its normal is perpendicular to $\mathbf{a}$ so $(a,0,c) \cdot (2,3,1) = 0 \Rightarrow 2a + c = 0$ choose $a = 1\, c = -2$

Thus the equation is $x - 2z = 0$

(iv) Let the equation be $ax + by + cz = k$.

The normal (a, b, c) is perpendicular to $\mathbf{a}$ and to $(3, -3, 0)$.

So $2a + 3b + c = 0$ and $3a - 3b = 0$

Thus $a = b\, c = -4b$ so choose $a = b = 1$ and $c = -4$

The equation therefore is $x + y - 4z = 1$ (contains (0,1,0))

(v) Let the equation be $ax + by + cz = k$ (as in (iv))

$$\left.\begin{array}{rcl} a + b + 2c & = & 0 \\ 3a - 3b & = & 0 \end{array}\right\} a = b = -c \text{ choose } a = b = -c = 1$$

So the plane is $x + y - z = 5$ as it contains (-1,2,-4)

(vi)

$$\begin{aligned} \cos\theta &= \frac{(9, 1, -4) \cdot (1, 1, -1)|}{|(1, 1, -4)||(1, 1, -1)|} \\ &= \frac{6}{\sqrt{18}\sqrt{3}} = \frac{2}{\sqrt{6}} \\ \theta &= 35^\circ \\ \text{or } \theta &= 0.6155 \text{ radians} \end{aligned}$$