Question

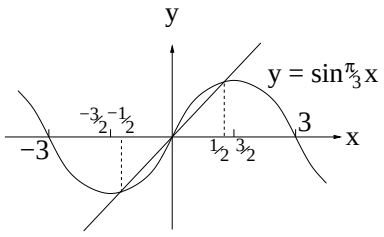
Let $f(x,y) = \left(\sin\frac{\pi}{3}x, \frac{y}{2}\right)$. Find all the fixed points and their stability. Give a sketch indicating basins of attraction and stable/unstable manifolds.

Answer

Fixed points: $\sin\frac{\pi}{3}x = x$, $\frac{y}{2} = y$ so $y = 0$ and $x = 0, \pm\frac{1}{2}$.

$DF(x,y) = \begin{pmatrix} \frac{\pi}{3}\cos\frac{\pi}{3}x & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$:

$DF(0,0) = \begin{pmatrix} \frac{\pi}{3} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$, $DF(\pm\frac{1}{2},0) = \begin{pmatrix} \frac{\pi}{2\sqrt{3}} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$.



Therefore saddle at (0,0), sinks at $(\pm\frac{1}{2},0)$.

Two basins of attraction:

$x > 0$(attracted to $(\frac{1}{2},0)$)

$x < 0$(attracted to $(-\frac{1}{2},0)$)