Question

Show that when a fixed point of the Henon map becomes unstable and creates a 2-cycle, then this occurs at the point $(x, y) = \left(\frac{1-b}{2}, \frac{1-b}{2} \right)$.

[Hint: Remember the expression for the sum of the roots of a quadratic equation.]

Answer

Every fixed point of f lies on the line $y = x$. Every 2-cycle $\{(x,y),(x',y')\}$ satisfies $x = x' = (1 - b)$, from the quadratic equation from question 2 whose roots are x, x'. At the moment the 2-cycle is created from a fixed point we have $x = x' = \frac{1}{2}(1 - b) = y = y'$.