Question

Find <u>three</u> 2-cycles and <u>three</u> 3-cycles for the hyperbolic toral automorphism given by the matrix $A = \begin{pmatrix} 3 & 7 \\ 2 & 5 \end{pmatrix}$. (There are many more!)

[<u>Hint</u>: find explicit solutions v to $A^n v = v \bmod 1$, for $n = 2,\ 3$.]

Answer

$(A^2 - I)^{-1} = \frac{1}{16} \begin{pmatrix} -12 & 8 \\ 8 & -4 \end{pmatrix}$ We apply this matrix to $\begin{pmatrix} k \\ l \end{pmatrix}$, $k, l \in \mathbf{Z}$.

$\left(\text{drop the } \frac{1}{30}\right)$

$\begin{pmatrix} k \\ l \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$: $\underline{\begin{pmatrix} -19 \\ 8 \end{pmatrix} \mapsto \begin{pmatrix} -1 \\ 2 \end{pmatrix} \mapsto \begin{pmatrix} -19 \\ 8 \end{pmatrix}}$

$\begin{pmatrix} k \\ l \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$: $\underline{\begin{pmatrix} 28 \\ -11 \end{pmatrix} \mapsto \begin{pmatrix} 7 \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} 28 \\ -11 \end{pmatrix}}$.

Add: $\underline{\begin{pmatrix} 9 \\ -3 \end{pmatrix} \mapsto \begin{pmatrix} 6 \\ 3 \end{pmatrix} \mapsto \begin{pmatrix} 9 \\ -3 \end{pmatrix}}$.

$(A^3 - I)^{-1} = \frac{1}{54} \begin{pmatrix} -34 & 14 \\ 49 & -20 \end{pmatrix}$:

points on <u>distinct</u> 3-cycles are $\frac{1}{54} \begin{pmatrix} -34 \\ 49 \end{pmatrix}, \begin{pmatrix} 14 \\ -20 \end{pmatrix}, \begin{pmatrix} -20 \\ 29 \end{pmatrix}$