QUESTION The set of all 2×2 matrices with real entries forms a vector space over $\mathbf{r}$. Which of the following subsets of matrices are subspaces?

(a) the set of those with zero trace;

(b) the set of those with zero determinant;

(c) the set of those with integer entries;

(d) the set of symmetric matrices.

ANSWER

(a) Yes - if $\mathrm{tr}A = \mathrm{tr}B = 0$ then $\mathrm{tr}(A+B) = \mathrm{tr}A + \mathrm{tr}B = 0$
and $\mathrm{tr}(\lambda A) = \lambda \mathrm{tr}A = 0$.

(b) No - it is easy to construct examples where $\det A = \det B = 0$ but $\det(A+B) \neq 0$.

(c) No - if A contains any odd numbers then $\frac{1}{2}A$ is not in the set.

(d) Yes - A and B symmetric imply both $A+B$ and λA symmetric.