

QUESTION Are the following structures vector spaces over  $\mathbf{R}$ ? If not, which axioms fail to hold?

(a) The set of triples of real numbers  $(x, y, z)$  with the operations

$$(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$$

$$\text{and } \lambda(x, y, z) = (0, 0, 0), \forall \lambda \in \mathbf{R}.$$

(b) The singleton set containing the planet Saturn with

$$\text{Saturn} + \text{Saturn} = \text{Saturn} \quad \text{and} \quad \lambda(\text{Saturn}) = \text{Saturn}, \forall \lambda \in \mathbf{R}.$$

ANSWER

(a) No - the axiom  $1(x, y, z) = (x, y, z)$  is violated here for most  $(x, y, z)$

(b) Yes.