

QUESTION

Evaluate $\int_{\alpha} \bar{z} dz$ and $\int_{\beta} \bar{z} dz$ where α is the contour defined by $z = e^{\pi it}$ for

$0 \leq t \leq \frac{1}{4}$ and β is the contour defined by $z = \begin{cases} t + it, & (0 \leq t \leq 1) \\ t + i, & (1 \leq t \leq 2) \end{cases}$

ANSWER

$$\int_{\alpha} \bar{z} dz = \int_0^{1/4} e^{-\pi it} \cdot \pi i e^{\pi it} dt = \pi it \Big|_0^{1/4} = i\pi/4.$$

$$\int_{\beta} \bar{z} dz = \int_0^1 (t - it)(1 + i) dt + \int_1^2 (t - i) dt = \frac{5}{2} - i$$