QUESTION

Find a parametrization of the contour that follows the real axis from 0 to 1 and then follows a straight line from 1 to $2 + i$.

ANSWER

For $0 \leq t \leq 1$, we put $z = t$. A parametrization of the line from 1 to $2 + i$ based on $[0, 1]$ is $(1 - s).1 + (2 + i)s$ To get a parametrization based on $[1, 2]$ we put $t = s + 1$ or $s = t - 1$. Then we get $2 - t + (2 + i)(t - 1) = t + i(t - 1)$. Thus the desired parametrization is

$$
\begin{cases}
z = t & 0 \leq t \leq 1 \\
z = t + i(t - 1) & 1 \leq t \leq 2.
\end{cases}
$$