QUESTION

The local supermarket in a small town buys and sells turkeys for Christmas. Each turkey costs the supermarket £5.00 and has to be ordered well in advance. They are sold to customers for £7.50. A week after Christmas, turkeys have to go on sale at half price. Based on previous experience, the owners of the supermarket know that the demand for turkeys over the Christmas period has the following distribution:

Demand	Probability
100	0.10
150	0.20
200	0.35
250	0.20
300	0.10
350	0.05

(i) Find an expression for the expected profit for a given order quantity.

(ii) Hence derive a method of finding the optimal order quantity.

(iii) Use your answer in part (ii) to determine how many turkeys the supermarket should order.

ANSWER

Inventories

Let $c = £5.00,\ s = £7.50,\ v = £3.75,\ d=$ demand and $q=$ quantity to be ordered. Then,

(i)

$$\text{Profit}(d,q) = sd + v(q-d) - cq \text{ if } d \le q$$
$$\text{Profit}(d,q) = sq - cq \text{ if } d > q$$

$$E(\text{Profit}) = \sum_{d=0}^{q}(sd+v(q-d)-cq)\text{Prob}(D=d)+\sum_{d=q+1}^{\infty}(sq-cq)\text{Prob}(D=d)$$

(ii) We then have

$$\begin{aligned} \text{Profit}(d,q) &= (v-c)q + (s-v)d \text{ for } d \le q & (1) \\ \text{Profit}(d,q) &= (s-c)q \text{ for } d > q & (2) \end{aligned}$$

For the situation represented by (1), ordering $q+1$ turkeys instead of q would decrease profit by an amount $(v-c)$. The probability of case (1) is $\text{Prob}(D \le q)$

In the situation represented by (2), ordering $q+1$ turkeys instead of q would increase profit by an amount of $(s-c)$. The happens with $\text{Prob}(D > q)$

If we call $E(q)$ the expected profit when ordering q turkeys, we see that

$$\begin{aligned} E(q+1) - E(q) &= (v-c)\text{Prob}(D \le q) + (s-c)\text{Prob}(D > q) \\ &= (v-c)\text{Prob}(D \le q) + (s-c)[1 - \text{Prob}(D \le q)] \\ &= (s-c) + (v-s)\text{Prob}(D \le q) \end{aligned}$$

Now, $E(q+1) - E(q) \ge 0$ if $(s-c) + (v-s)\text{Prob}(D \le q) \ge 0$ therefore ,

$$\text{Prob}(D \le q) \le \frac{s-c}{s-v} \qquad (3)$$

and we look for the largest value of q such that (3) holds.

(iii) We need $\sum_{d=1}^{q} \text{Prob}(D = d) \le \frac{7.50 - 5.00}{7.50 - 3.75} = 0.67$

$$\text{Prob}(D = 100) + \text{Prob}(D = 150) + \text{Prob}(D = 200) = 0.65$$

The supermarket should order 200 turkeys.