QUESTION

**(a)** A project consists of activities $A, B, \ldots, J$ whose prerequisites are given in the table below. Draw a network, suitable for analysis by the critical path method, to represent the project. You should avoid using dummy activities, where possible. For each event, write the earliest and latest event times on the network and deduce the critical path.

| Activity | Prerequisites | Duration(days) |
|---|---|---|
| $A$ | - | 8 |
| $B$ | - | 9 |
| $C$ | $A$ | 7 |
| $D$ | $A$ | 4 |
| $E$ | $B$ | 6 |
| $F$ | $B, D$ | 1 |
| $G$ | $B, D$ | 1 |
| $H$ | $C, F$ | 7 |
| $I$ | $E, G$ | 4 |
| $J$ | $E$ | 8 |

If it is possible to transfer resources from activity $E$ to $H$ so that the duration of $H$ is reduced by one day but the duration of $E$ is increased by one day, decide, giving a reason, if the project duration can be reduced.

**(b)** A doctor diagnoses that a patient has either disease $A$ or disease $B$, and that on the current evidence these two alternatives are equally likely.

Although there is no test for disease $B$, it is possible to undertake an expensive test for disease $A$. A disadvantage of this test is unreliability. In particular, it gives a positive indication of disease $A$ for only 70% of patients who have disease $A$, and also gives a positive indication of disease $A$ for 10% of patients who have disease $B$ rather than $A$.

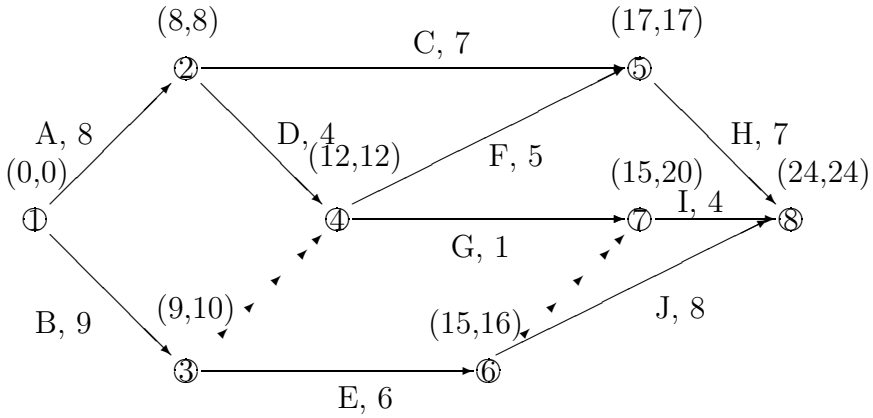
There is no effective treatment for disease $B$, and patients with this disease remain in poor health thereafter. Similarly, if patients suffering from disease $A$ go untreated, then they also remain in poor health. However, there is an expensive treatment for disease $A$. A patient with disease $A$ receiving treatment is returned to good health with probability 0.7, and remains in poor health with probability 0.3. Using this treatment on a patient with disease $B$ has no effect.

The utility of a patient in good health is 30, and of a patient in poor health is 10. Utilities are reduced by 2 if a patient undergoes the test, and are reduced by 1 if a patient receives the treatment.

Develop a decision tree to find out whether the patient should undergo the test, and whether the treatment for disease $A$ should be administered.

ANSWER

**(a)**




The critical path is $A$ - $D$ - $F$ - $H$

$TF_E = 16 - 9 - 6 = 1$

Since $H$ is not a successor of $E$, there is a path containing $E$ that becomes critical when the duration of $E$ is increased. Therefore, the project duration is not reduced.

**(b)** Let $T$ denote that the test is undertaken.

$\overline{T}$ denote that the test is not undertaken

$P$ denote a positive indication of disease $A$

$\overline{P}$ denote no positive indication of disease $A$

$H$ denote that the patient returns to good health

$\overline{H}$ denote that the patient remains in poor health.

We are told that $P(A) = P(B) = 0.5$

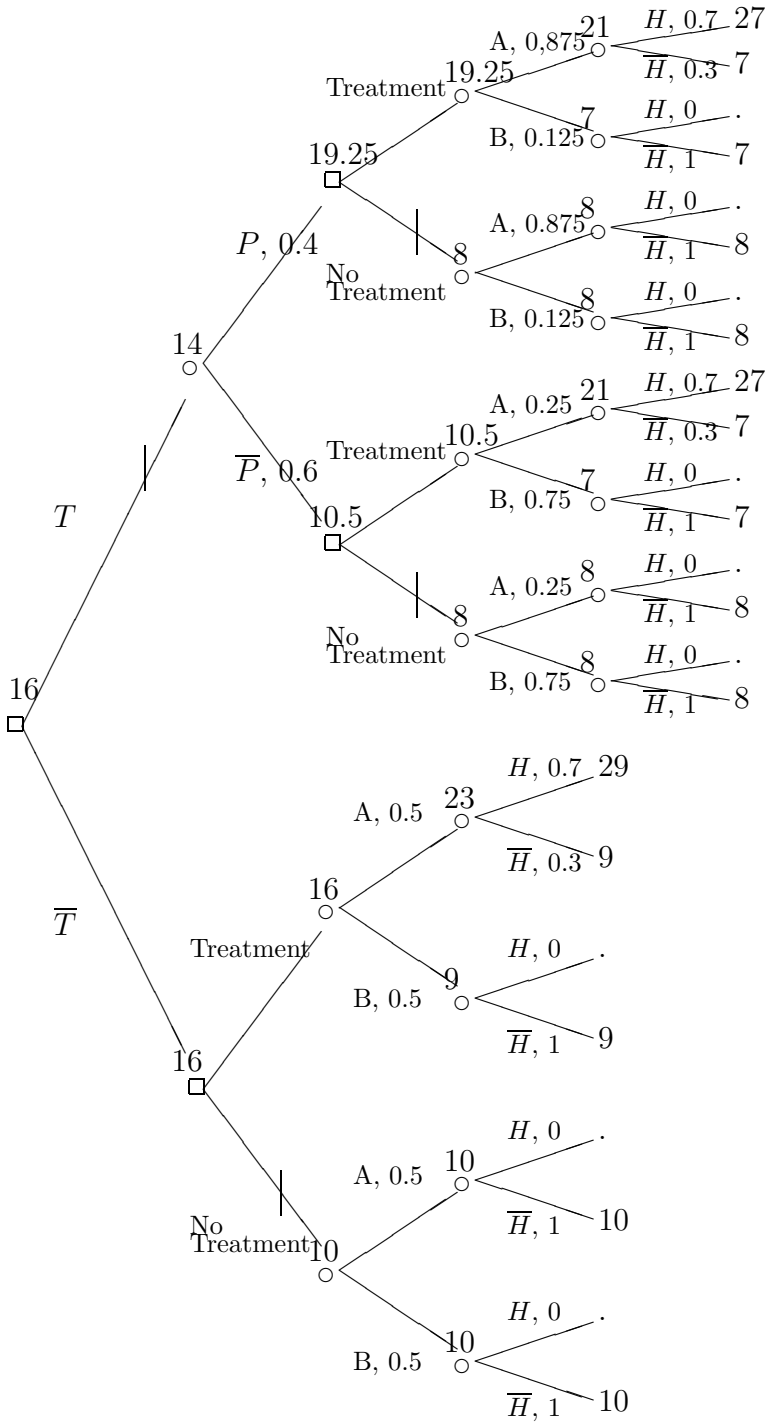
$$P(P|A) = 0.7 \quad P(\overline{P}|A) = 0.3$$
$$P(P|B) = 0.1 \quad P(\overline{P}|B) = 0.9$$

$$\begin{aligned} P(P) &= P(P|A)P(A) + P(P|B)P(B) = 0.4 \\ P(\overline{P}) &= P(\overline{P}|A)P(A) + P(\overline{P}|B)P(B) = 0.6 \end{aligned}$$

The posterior probabilities are

$$\begin{aligned} P(A|P) &= \frac{P(P|A)P(A)}{P(P)} = 0.875 \\ P(B|P) &= \frac{P(P|B)P(B)}{P(P)} = 0.125 \\ P(A|\overline{P}) &= \frac{P(\overline{P}|A)P(A)}{P(\overline{P})} = 0.25 \\ P(B|\overline{P}) &= \frac{P(\overline{P}|B)P(B)}{P(\overline{P})} = 0.75 \end{aligned}$$

The decision tree is given below




It is recommended that the treatment should be administered, but the test should not be performed.