Question
Let $\mathcal{R}[0,1]$ be the set of Riemann-integrable functions over [0,1]. Find a sequence of functions $\{f_n\}$, $f_n : [0,1] \rightarrow \mathbf{R}$, with the properties

i) there exists $f : [0,1] \rightarrow \mathbf{R}$, for all $x \epsilon [0,1]$, $\lim_{n\rightarrow\infty} f_n(x) = f(x)$

ii) there exists $M \epsilon \mathbf{R}$, for all $n \epsilon \mathbf{N}$, for all $x \epsilon [0,1]$, $|f_n(x)| \leq M$

iii) for all $x \epsilon [0,1]$ $|f(x)| \leq M$

iv) for all $n \epsilon \mathbf{N}$, $f_n \epsilon \mathcal{R}[0,1]$

v) $f \not\epsilon \mathcal{R}[0,1]$

and summarise these properties in words.

Answer
Let $r_1, r_2 \cdots$ be an enumeration of the rationals in $[0,1]$.
Let $f_n(x) = \begin{cases} 1 & \text{if } x = r_1, r_2, \cdots r_n \\ 0 & \text{otherwise} \end{cases}$
This provides the required example of a uniformly bounded sequence of Riemann integrable functions converging to a non-Riemann integrable function.