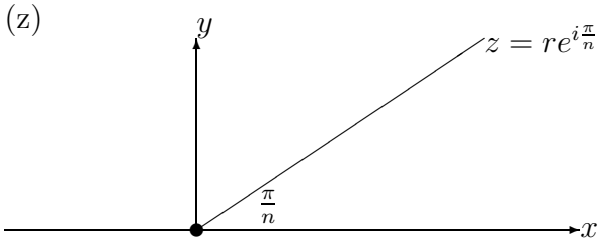
Question

What is the image of the wedge defined by $0 \leq \arg z \leq \dfrac{\pi}{n},\ n$ integer, under the transformation $w = f(z) = z^m,\ m$ integer? Discuss any special values of m. What happens if n or m is non-integer? What happens when $m > 2n$?

Answer

Define wedge boundaries by $z = re^{i\frac{\pi}{n}}$ and $z = r,\ r > 0$.

(z)

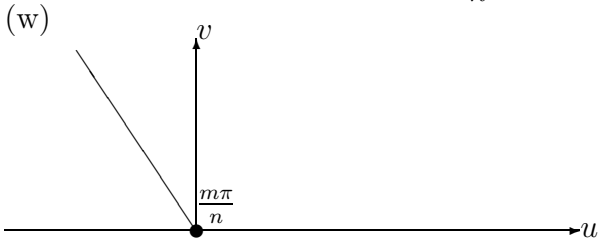


Then $w = f(z) = z^m$ gives

$$w = r^m e^{im\frac{\pi}{n}},\ z = r^m$$

i.e., a wedge of angle of opening $\dfrac{m\pi}{n}$:

(w)

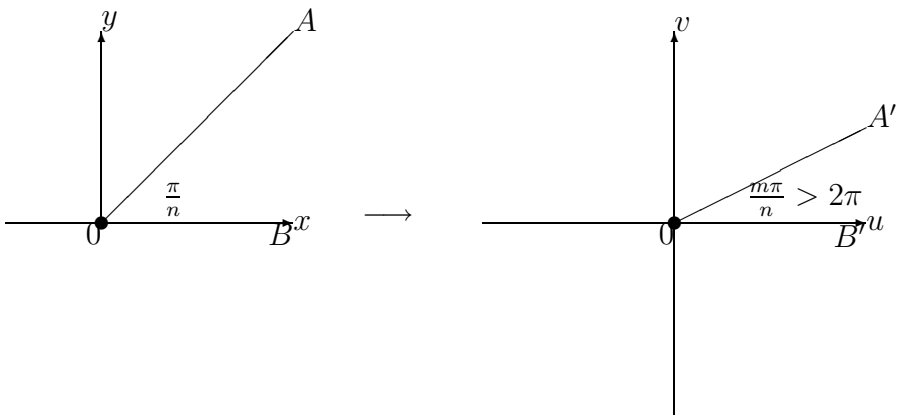


Note that if $m = n$ we have wedge in $z \longrightarrow$ upperhalf plane in w.

If $m = 2n$ we have wedge in $z \longrightarrow$ complete w-plane.

If m or n is non-integer, we just have an irrational "fraction" of π as an opening angle of the sector in (w).

If $m > 2n$ we map the z-wedge onto more than one revolution of the (w) plane.



This is bad, since it can lead to ambiguities, i.e., certain values of w refer to <u>two</u> values of z (in the overlap region of w-plane).