Question

On board the Cutty Sark at Greenwich one can see the tonnage calculation done in 1869 by the ship's designer Hercules Linton (not Brunel!). This involves working out the areas of eleven equally spaced cross sections of the hull. For each one, Linton divided half the cross section unto six horizontal strips of equal width, and then used Simpson's rule. At the largest cross section the depth of the hull is 20.4 feet, and its half-widths at the seven levels are 17.15, 17.40, 17.45, 17.30, 16.10, 10.65, 0.60 feet.

Show that the area of the cross section is just over 600 square feet. If you knew all eleven cross-sectional areas, how would you calculate the volume if the hull, given that the length of the Cutty Sark's hull is 213 feet?

PICTURE

Answer

For those of you who don't know, the Cutty Sark is a ship! It was a Tea Cutter and was one of the fastest ships of its day (19th century). It's also the name of a good pub on the waterfront at Greenwich, but that's another story...

We must use Simpson's rule with 7 ordinates (with 6 strips). Look at the diagram on the sheet and we have $h = \frac{204}{6}$ with

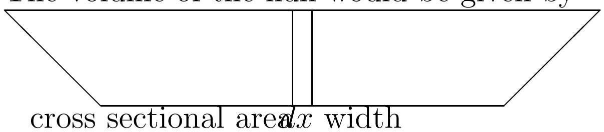
$y_1 = 17.15,\ y_2 = 17.40,\ y_3 = 17.45,\ y_4 = 17.30,\ y_5 = 16.10,\ y_6 = 10.65,\ y_7 = 0.60$

Simpson with 7 ordinates gives

$$\begin{aligned} \text{Area} &= \frac{20.4}{6\times 3}(\underbrace{17.15+0.60}_{y_1+y_7} + 4\times(\underbrace{17.40+17.30+10.65}_{y_2+y_4+y_6}) \\ &\quad +2\times(\underbrace{17.45+16.10}_{y_3+y_5})) \\ &= 301.75 \text{ square feet} \end{aligned}$$

This is only $\frac{1}{2}$ the area of cross-section (see diagram on sheet) so total cross-sectional area=$603.5 \approx 600$ square feet.

The volume of the hull would be given by



volume=$\int_0^L$(area of elemental cross section)$\times\, dx$

where dx =width of elemental cross-section, L=length of strip.

With only 11 cross-sectional areas we approximate this by Simpson's rule with 11 ordinates, 10 strips. Hence here $h = \dfrac{213}{10} = 21.3$, y_i =area of section i

volume $\approx \dfrac{h}{3}(y_1 + 4y_2 + 2y_3 + 4y_4 + 2y_5 + 4y_6 + 2y_7 + 4y_8 + 2y_9 + 4y_{10} + y_{11})$