Question

Find, to 3 D.P., the error involved in estimating $\int_0^4 \dfrac{dx}{1+x}$ by using Simpson's rule with five ordinates.

Answer

First the actual value of the integral:

$$I = \int_0^4 \frac{dx}{1+x} = [\ln(1+x)]_0^4 = \ln 5 = \underline{1.609} \text{ to 3d.p.}$$

Simpson's rule with 5 ordinates $\Rightarrow$ 4 strips $\Rightarrow h = \dfrac{4-0}{4} = 1$

Thus $y = \dfrac{1}{1+x}$

x	0	1	2	3	4
y	1.000	0.500	0.333	0.250	0.200

So

Area

$= \dfrac{h}{3}(y_1 + 4y_2 + 2y_3 + 4y_4 + y_5)$

$= \dfrac{1}{3} \times (\underbrace{1.000 + 0.200}_{y_1 + y_5} + 4(\underbrace{0.500 + 0.250}_{y_2 + y_4}) + 2 \times \underbrace{0.333}_{y_3})$

$\underline{1.622}$ to 3d.p.

Thus error= $|1.609 - 1.622| = \underline{0.013}$