Question Calculate $\int_1^4 \log_{10} x \, dx$ to 4 S.F. using

(i) the trapezium rule with seven ordinates,

(ii) Simpson's rule with seven ordinates.

Answer

(i) Trapezium rule with 7 ordinates.

$$\text{Area} \approx \frac{d}{2}(y_1 + 2y_2 + 2y_3 + 2y_4 + 2y_5 + 2y_6 + y_7)$$

Divide range into 6 equal segments $\Rightarrow d = \frac{(4-1)}{6} = \frac{1}{2} = 0.5$

$y = \log_{10} x$

x	1	1.5	2.0	2.5	3.0	3.5	4.0
y	0.000	0.1761	0.3010	0.3979	0.4771	0.5441	0.6021

$$\begin{aligned}\text{Area} &= \frac{0.5}{2}(\underbrace{0.000 + 0.6021}_{y_1 + y_7} \\ &\quad +2\underbrace{(0.1761 + 0.3010 + 0.3979 + 0.4771 + 0.5441)}_{y_2 + y_3 + y_4 + y_5 + y_6}) \\ &= \underline{1.0986 = 1.099\text{to 4s.f.}}\end{aligned}$$

(ii) Simpson's rule with 7 ordinates

$$\text{Area} = \frac{h}{3}(y_1 + 4y_2 + 2y_3 + 4y_4 + 2y_5 + 4y_6 + y_7)$$

Again divide into 6 equal segments $\Rightarrow h = 0.5$ again and the x and y values are identical to part (i), so we have

$$\begin{aligned}\text{Area} &= \frac{0.5}{3}(\underbrace{0.000 + 0.6021}_{y_1 + y_7} + 4\underbrace{(0.1761 + 0.3979 + 0.5441)}_{y_2 + y_4 + y_6} \\ &\quad +2(\underbrace{0.3010 + 0.4771}_{y_3 + y_5})) \\ &= \underline{1.10512 = 1.105\text{to 4s.f.}}\end{aligned}$$

[Actual value $= \frac{1}{\ln 10} + \frac{4(\ln 4 - 1)}{\ln 10} = 1.105$to 4s.f.]

So Simpson's rule is more accurate.