Question

Calculate $\displaystyle\int_0^2 \frac{dx}{1+x^3}$ to 3 S.F. using

(i) the trapezium rule with five ordinates,

(ii) Simpson's rule with five ordinates.

Answer

(i) Trapezium rule with 5 ordinates

Area $\approx \dfrac{d}{2}(y_1 + 2y_2 + 2y_3 + 2y_4 + y_5)$

Range of integration $= 0 \to 2 \Rightarrow$ divide into $\underline{4}$ equal segments $\Rightarrow d = \dfrac{2}{4} = \dfrac{1}{2} = 0.5$

$y = \dfrac{1}{1+x^3}$

x	0	0.5	1.0	1.5	2.0
y	1.000	0.889	0.500	0.229	0.111

$$\begin{aligned} \text{Area} &= \frac{0.5}{2}(\underset{y_1}{1.000} + \underset{y_5}{0.111} + 2\underbrace{(0.889 + 0.500 + 0.229)}_{y_2+y_3+y_4}) \\ &= \underline{1.087 = 1.09 \text{to 3s.f.}} \end{aligned}$$

(ii) Simpson's rule with 5 ordinates

Area $= \dfrac{h}{3}(y_1 + 4y_2 + 2y_3 + 4y_4 + y_5)$

Again divide into 4 equal segments $\Rightarrow h = 0.5$

Thus we will have the same x and y values. So we can use the table above.

$$\begin{aligned} \text{Area} &= \frac{0.5}{3} \times (\underbrace{1.000 + 0.111}_{y_1+y_5} + 4(\underbrace{0.889 + 0.229}_{y_2+y_4}) + 2 \times \underset{y_3}{0.500}) \\ &= \underline{1.097 = 1.10 \text{to3s.f.}} \end{aligned}$$

[Actual value $= \dfrac{\pi}{2\sqrt{3}} + \dfrac{\log 3}{6} = \underline{1.09 \text{to 3s.f.}}$]

Note that Simpson's rule isn't so good here due to rounding errors.