Question

In a certain AC circuit, the applied voltage is $V = V_0 \sin \omega t$ volts. The corresponding current is $I = V_0 \omega C \cos \omega t$ amps. Given that the power dissipated P is given by $P = IV$. Find the mean value of P over a period. Find the corresponding rms value.

Answer

$P + IV + V_0 \sin \omega t \times V_0 \omega c \cos \omega t$

(The period will be from $t = 0$ to $t = \dfrac{2\pi}{\omega}$)

So

$$\begin{aligned} P &= V_0^2 \omega c \sin \omega t \cos \omega t \\ &= \frac{V_0^2 \omega c}{2} \sin 2\omega t \end{aligned}$$

Mean value of P over a period is

$$\begin{aligned} &= \frac{1}{(\frac{2\pi}{\omega} - 0)} \int_0^{\frac{2\pi}{\omega}} \frac{V_0^2 \omega c}{2} \sin 2\omega t \, dt \\ &= \frac{\omega}{2\pi} \frac{V_0^2 \omega c}{2} \left[-\frac{cos 2\omega t}{2\omega} \right]_0^{\frac{2\pi}{\omega}} \\ &= \underline{0} \end{aligned}$$

rms value over a period

$$\begin{aligned} &= \sqrt{ds \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} P^2 \, dt} \\ &= \left[\frac{\omega}{2\pi} \frac{V_0^4 \omega^2 c^2}{4} \int_0^{\frac{2\pi}{\omega}} \sin^2 2\omega t \, dt \right]^{\frac{1}{2}} \\ &= \frac{V_0^2 \omega^{\frac{3}{2}} c}{2\sqrt{2\pi}} \left\{ \int_0^{\frac{2\pi}{\omega}} \sin^2 2\omega t \, dt \right\}^{\frac{1}{2}} \\ & \quad 1 - 2\sin^2 2\omega t = \cos 4\omega t \\ &= \frac{V_0^2 \omega^{\frac{3}{2}} c}{2\sqrt{2\pi}} \left\{ \frac{1}{2} \int_0^{\frac{2\pi}{\omega}} (1 - \cos 4\omega t) \, dt \right\}^{\frac{1}{2}} \\ &= \frac{V_0^2 \omega^{\frac{3}{2}} c}{4\sqrt{\pi}} \left\{ \left[t - \frac{\sin 4\omega t}{4\omega} \right]_0^{\frac{2\pi}{\omega}} \right\}^{\frac{1}{2}} \\ &= \underline{\frac{V_0^2 \omega c}{2\sqrt{2}}} \end{aligned}$$