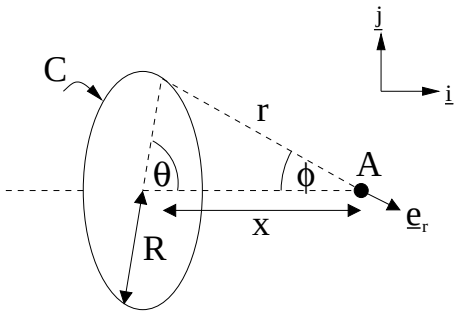
Question
Find the gravitational field along the axis of a uniform density circular hoop as a function of distance from the centre of the hoop.

Answer



The gravitational field at A is parallel to the axis (by symmetry). Therefore the only component we have to consider is the **i** component.

$$
\begin{aligned}
&= -Gm \int_C \frac{\rho}{r^2} \cos\phi \, ds \\
&= -Gm\rho \int_{\theta=0}^{\theta=2\pi} \frac{\cos\phi}{r^2} R \, d\theta \quad \text{as } ds = r \, d\theta \\
&= -Gm\rho \frac{\cos\phi}{r^2} R \times 2\pi \\
&= -\frac{2\pi Gm\rho R x}{(R^2+x^2)^{\frac{2}{3}}} \\
&\quad \text{Since } r^2 = R^2 + x^2 \text{and } \cos\phi = \frac{x}{\sqrt{r^2+x^2}}
\end{aligned}
$$