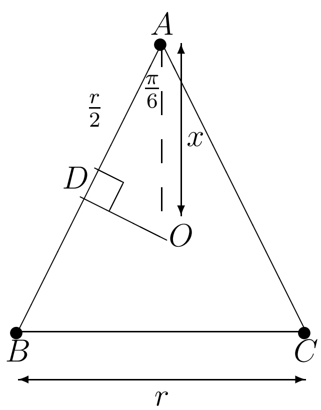
Question

Suppose that three particles of equal mass m are placed at the corners of an equilateral triangle of side length d. Suppose that these three particles subsequently move from rest under the influence of their mutual gravitational forces. Find their speed when they have moved a distance $\frac{d}{2}$.

Answer



From triangle ADO $x\cos\dfrac{\pi}{6} = \dfrac{r}{2} \quad \Rightarrow x\sqrt{3} = r$

The force on A is directed along AO, and it has magnitude $\sqrt{3}\dfrac{Gm^2}{r^2}$

$\Rightarrow m\ddot{x} = \sqrt{3}\dfrac{Gm^2}{r^2} \quad \Rightarrow \ddot{x} = -\dfrac{\alpha}{x^2} \quad$ where $\alpha = \dfrac{Gm}{\sqrt{3}}$

Now $\dfrac{d^2x}{dt^2} = v\dfrac{dv}{dx} \quad \Rightarrow \dfrac{dv}{dx} = -\dfrac{\alpha}{x^2} \quad \Rightarrow \dfrac{1}{2}v^2 = \dfrac{\alpha}{x} + A \quad (*)$

The initially conditions are $v = 0, \quad r = d \quad \Rightarrow x = \dfrac{d}{\sqrt{3}}$

Putting this into equation * gives $0 = \dfrac{\alpha}{\frac{d}{\sqrt{3}}} + A \quad \Rightarrow A = -\dfrac{\alpha\sqrt{3}}{d}$

$$\text{therefore} \quad \frac{1}{2}v^2 = \alpha\left(\frac{1}{x} - \frac{\sqrt{3}}{d}\right)$$

At $x = \dfrac{d}{2}, \quad \dfrac{1}{2}v^2 = \alpha\left(\dfrac{2}{d} - \dfrac{\sqrt{3}}{d}\right) \Rightarrow v = \sqrt{\dfrac{2\alpha}{d}\left(2 - \sqrt{3}\right)}$