Question

Find the solution of the initial value problem
$$\frac{dx}{dt} = \frac{2}{3}\frac{1}{x^2} = \frac{2}{3}t \times \frac{1}{x^2}; \text{ with } x(0) = 0$$

Answer

$$\frac{dx}{dt} = \frac{2}{3}\frac{1}{x^2} = \frac{2}{3}t \times \frac{1}{x^2}$$

Separable:

$$
\begin{aligned}
\Rightarrow x^2\, dx &= \frac{2}{3}t\, dt \\
\Rightarrow \int x^2\, dx &= \frac{2}{3}\int t\, dt \\
\Rightarrow \frac{1}{3}x^3 &= \frac{1}{3}t^2 + \text{constant} \\
\Rightarrow x^3 &= t^2 + \text{constant}
\end{aligned}
$$

Now $x(0) = 0$, so constant is zero

Solution is $x^3 = t^2 \Rightarrow \quad x = t^{\frac{2}{3}}$