Question

Classify, but do not solve, the equation $t^2 \dfrac{dx}{dt} + 2xt = 0$
under as many as possible of the following classifications:

$$\text{separable, } \tfrac{dx}{dt} = f\left(\tfrac{x}{t}\right), \text{ exact, linear.}$$

Answer

$$t^2 \frac{dx}{dt} + 2xt = 0$$

$$t^2 \frac{dx}{dt} + 2xt = \frac{d}{dt}(xt62) \Rightarrow \text{ exact No powers of } x^2 \text{ or } \left(\tfrac{dx}{dt}\right) \text{ etc} \Rightarrow \text{ linear.}$$

Equation can be rewritten as $\frac{dx}{dt} = -2\frac{x}{t} \Rightarrow f(x,t) = f(\tfrac{x}{t}) = -2\frac{x}{t}$

$$t^2 \frac{dx}{dt} + 2xt = 0 \Rightarrow \frac{dx}{dt} = (-2x)\left(\frac{1}{t}\right) = g(x)h(t) \Rightarrow \text{Separable}$$