Question

The following equations are written in terms of cylindrical co-ordinates (ρ, ϕ, z). What surfaces or curves do they represent?

(a) $\phi = \frac{\pi}{4}$, $z = 2$;

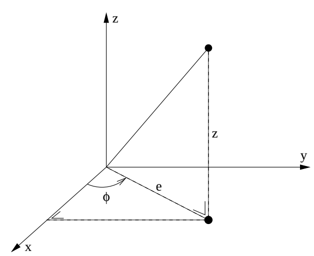
(b) $\rho^2 + z^2 = 9$;

(c) $\rho = z\tan(\alpha)$ where $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ is a real constant;

(d) $\rho\sin(\phi) = 1$, $z = 0$.

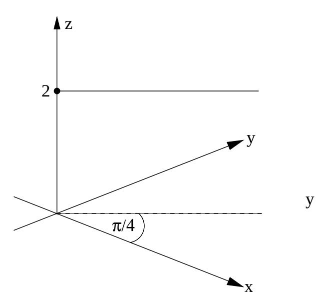
Answer

Cylindrical co-ordinates (ρ, ϕ, z).
$\rho \geq 0$ and $0 \leq \phi \leq 2\pi$
Also, $x = \rho\cos\phi$ and $y = \rho\sin\phi$



(a)

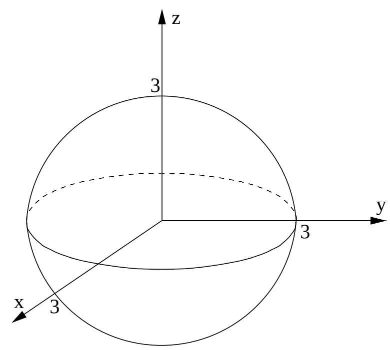
$\phi = \frac{\pi}{4}$ and z = 2
This gives a half line at height $z = 2$ in the direction $\phi = \frac{\pi}{4}$
(i.e. x = y)



(b) $\rho^2 + z^2 = 9$

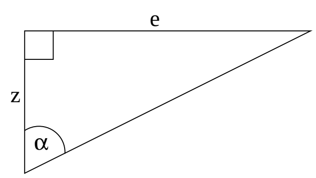
Now $x^2 + y^2 = \rho^2\cos^2\phi + \rho^2\sin^2\phi = \rho^2$

So we have $(x^2 + y^2) + z^2 = 9$ or $x^2 + y^2 + z^2 = 3^2$ which defines a sphere centre the origin of radius 3.



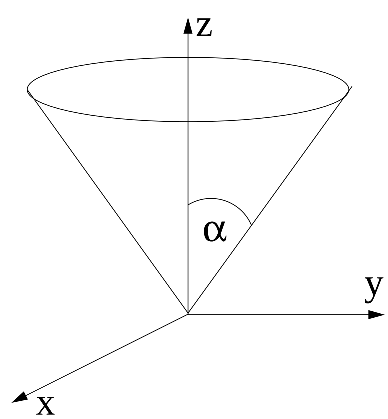
(c) $\rho = z\tan(\alpha)$ for $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$

we have the righthanded triangle

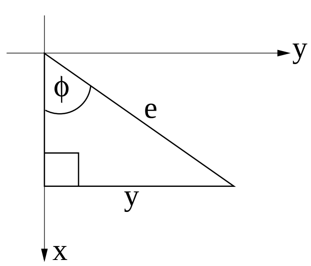


so that $\tan(\alpha) = \frac{\rho}{z}$
and hence $\rho = z\tan(\alpha)$.

If we now let ϕ vary as $0 \le \phi \le 2\pi$, we obtain a cone angle α.



(d) $\rho\sin(\phi) = 1$, $z = 0$. Since $z = 0$ we restrict to the xy plane.



Now from the triangle we have
$\sin\phi = \frac{y}{\rho}$
and so $y = \rho\sin\phi$.

Hence $\rho \sin \phi = 1 \Rightarrow y = 1$, and letting the x vary we obtain the line $y = 1$ in the xy-plane.