QUESTION Use the standard semicircular contour to show that

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{ab(a+b)}, \;\; a, b > 0$$

ANSWER

$$I = \int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)} = 2\pi i(\mathrm{Res}(ia) + \mathrm{Res}(ib))$$

This has 4 poles, 2 inside the contour.

$\mathrm{Res}(z_0) = \dfrac{1}{2z_0(z_0^2+b^2) + 2z_0(z_0^2+a^2)}$

$I = 2\pi i \left(\dfrac{1}{2ia(b^2-a^2)} + \dfrac{1}{2ib(a^2-b^2)} \right) = \dfrac{\pi(b-a)}{ab(b^2-a^2)} = \dfrac{\pi}{ab(a+b)}$