QUESTION

Use the standard semicircular contour to evaluate

(a)

$$\int_0^\infty \frac{dx}{2+x^2}$$

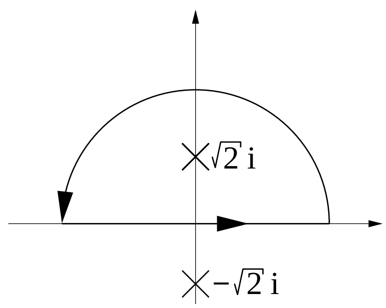
(b)

$$\int_0^\infty \frac{x^2dx}{\left(x^2+a^2\right)^2}$$

ANSWER

(a)

$$I = \frac{1}{2}\int_{-\infty}^{\infty} \frac{d}{2+x^2}$$

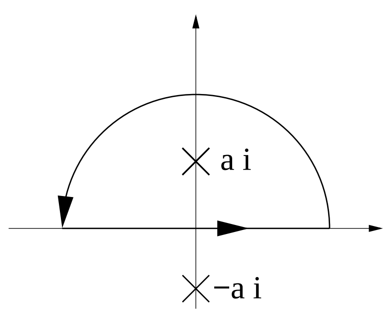


This has two simple poles, 1 inside the contour.

$I = \frac{1}{2}2\pi i \frac{1}{2\sqrt{2}i} = \frac{\pi}{2\sqrt{2}}$

(b)

$$I = \frac{1}{2}\int_{-\infty}^{\infty} \frac{x^2dx}{\left(x^2+a^2\right)^2}$$



This has two double poles, 1 inside the contour.

$$\text{Res}\left(\frac{z^2}{\left(z^2+a^2\right)^2}, a_i\right) = \lim_{z\to ai} \frac{d}{dz}\frac{z^2}{\left(z+ai\right)^2}$$

$$
\begin{aligned}
&= \lim_{z\to a_i}\left(\frac{2z}{(z+ai)^2} - \frac{2z^2}{(z+ai)^3}\right) \\
&= \frac{2ai}{-4a^2} - \frac{-2a^2}{-8a^3 i} = -\frac{i}{4a} \\
\text{So } I &= \frac{1}{2}2\pi i\left(-\frac{1}{4a}\right) = \frac{\pi}{4a}
\end{aligned}
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