QUESTION

(a) Compute the following products of the permutations $\sigma = (1576)(234)$, $\tau = (132)(4675)$ expressing your answers in disjoint cycle notation:

(i) $\sigma\tau$.

(ii) $\tau\sigma$.

(iii) $\sigma^2\tau^{-1}$.

(b) Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 6 & 1 & 3 & 7 & 5 & 2 \end{pmatrix}$ in disjoint cycle notation and as a product of transpositions. Fin the order and sign of σ and calculate σ^{2001}.

(c) List all the possible cycle structures for elements of S_7 and use this to find all the possible orders for elements of S_7.

(d) Find all the possible cycle structures corresponding to elements of order 6 in S_9, and compute the number of elements of S_9 corresponding to each cycle structure.

ANSWER

(a) **(i)** $\sigma\tau = (14)(25)$

(ii) $\tau\sigma = (14)(36)$

(iii) $\sigma^2\tau^{-1} = (14637)$

(b) $\sigma = (143)(2657) = (14)(43)(26)(65)(57)$ has order 12 and sign -1. $2001 = 166.12 + 8$ so $\sigma^{2001} = \sigma^9$. Now σ^9 generates the same subgroup of $\langle\sigma\rangle$ as does σ^3, hence it has order 4.

(c)

Cycle structure	order
[7]	7
[6]	6
[5, 2]	10
[5]	5
[4, 3]	12
[4, 2]	4
[4]	4
[3, 3]	3
[3, 2, 2]	6
[3, 2]	6
[3]	6
[2, 2, 2]	2
[2, 2]	2
[2]	2
[1]	1

(d) The possible cycle structures are $[3,3,2]$, $[3,2,2]$, $[3,2]$, $[6,3]$, $[6,2]$, $[6]$. There are respectively 9.8.7.6.5.4.3.2/(3.3.2.2), 9.8.7.6.5.4.3/(3.2.2.2), 9.8.7.6.5/(3.2), 9.8.7.6.5.4.3.2.1/(6.3), 9.8.7.6.5.4.3.2/(6.2) and 9.8.7.6.5.4/6 elements with these structures.