Exam Question
Topic: Double Integral
Evaluate the double integral

$$\iint_R x^2 \sin\left(x^4 + 2x^2y^2 + y^4\right) d(x, y),$$

where R is the region satisfying $x^2 + y^2 \leq 1$ and $y \geq 0$.

Solution
In polar coordinates

$$
\begin{aligned}
I &= \int_0^\pi d\theta \int_0^1 r^2 \cos^2\theta \sin(r^4)\, dr \\
&= \int_0^\pi \cos^2\theta\, d\theta \int_0^1 r^3 \sin(r^4)\, dr \\
&= \frac{1}{2}\left[\theta + \frac{\sin\theta}{2}\right]_0^\pi \frac{1}{4}\left[-\cos(r^4)\right]_0^1 \\
&= \frac{\pi}{2}\cdot\frac{1}{4}(1 - \cos 1)
\end{aligned}
$$