Exam Question
Topic: Double Integral
The region R in the x-y plane is specified by the conditions

$$1 \leq x^2 + y^2 \leq 9, \ x \geq 0.$$

Evaluate the double integral

$$\iint_R \sin\left(\frac{\pi(x^2 + y^2)}{4}\right) d(x, y).$$

Solution
This integral is clearly to be done in polar coordinates

$$\int_{\theta=-\pi/2}^{\pi/2} d\theta \int_{r=1}^{3} \sin\left(\frac{\pi(x^2 + y^2)}{4}\right) .r \, dr$$

$$= \pi \left[-\frac{2}{\pi} \cos\left(\frac{\pi r^2}{4}\right)\right]_1^3 = 2\left[-\cos\left(\frac{9\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right)\right] = 0.$$