

Exam Question**Topic: Double Integral**

The region R is a trapezium bounded by the lines $x = \frac{1}{2}$, $x = 1$, $y = x$, $y = 2x$.

Evaluate the double integral

$$\iint_R \frac{\sin x}{x} d(x, y).$$

Give your answer in exact form and also as an approximation rounded to four decimal places using your calculator.

Solution

To evaluate the double integral we integrate with respect to y first, giving:

$$\begin{aligned} \int_{x=1/2}^1 dx \int_{y=x}^{2x} \frac{\sin x}{y} dy &= \int_{x=1/2}^1 \left[\frac{y \sin x}{x} \right]_{y=x}^{2x} dx \\ &= \int_{x=1/2}^1 \sin x dx = \cos(1/2) - \cos(1) = 0.3373 \quad (4 \text{ d.p.}) \end{aligned}$$